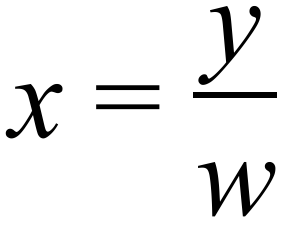
|  |  |
| --- | --- |
|  | **MATHEMATICS:SPECIALIST 1 & 2**  **SEMESTER 1 2019**  **TEST 1**  **Calculator Free** |

Reading Time: 2 minutes

Time Allowed: 30 minutes Total Marks: 29

Question 1 (3 marks)

Write the mathematical notation for the statement:

*For all rational numbers x, there exist integers y and w such that  where w is non-zero.*



Question 2 (11 marks)

Three vectors are given by , and .

(a) Determine

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states vector |

(ii) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states exact value |

(iii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines scalar multiples  ✓ determines sum |

(b) Determine the unit vector that is parallel and in the same direction as . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines  ✓ determines magnitude  ✓ states unit vector |

(c) Express in terms of and . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equates -coeffs  ✓ equates -coeffs  ✓ solves equations for first variable  ✓ solves equations for second variable and states |

Question 3 (8 marks)

(a) Write the inverse of the following true statement and comment on the truth of the inverse statement. (2 marks)

"If the discriminant of the quadratic formula is zero, then the quadratic has just one real root."

|  |
| --- |
| **Solution** |
| If the discriminant of the quadratic formula is **not** zero, then the quadratic **does not have** just one real root.  Statement is true. |
| **Specific behaviours** |
| ✓ changes 'if P then Q' to 'if not P then not Q'  ✓ indicates statement is true |

(b) Write the converse of the following true statement and comment on the truth of the converse statement. (2 marks)

"If then ."

|  |
| --- |
| **Solution** |
| If then .  Statement is false. |
| **Specific behaviours** |
| ✓ changes 'if P then Q' to 'if Q then P'  ✓ indicates statement is false |

(c) Determine the truth of the following statements, using an example or counter-example to support each answer.

(i) If and is an even number then is an even number. (2 marks)

|  |
| --- |
| **Solution** |
| Statement is false.  If (even) then (irrational, not even). |
| **Specific behaviours** |
| ✓ states false  ✓ supplies counter-example |

(ii) If and then . (2 marks)

|  |
| --- |
| **Solution** |
| Statement is false.  If then but . |
| **Specific behaviours** |
| ✓ states false  ✓ supplies counter-example using integers |

Question 4 (7 marks)

(a) A body moves from to .

(i) Determine the displacement vector in component form. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expresses in component form |

(ii) Determine the magnitude of the vector . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states magnitude |

(b) A force of N acts on a body. Determine the magnitude of the force and the angle its direction makes with the positive -axis. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states magnitude  ✓ states angle |

(c) A body moves with a velocity of 20 ms-1 at an angle of 135° with the positive -axis. Express the velocity of the body in the form , where and are constants.

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines expressions for and  ✓ simplifies and  ✓ states in required form |

|  |  |
| --- | --- |
|  | **MATHEMATICS:SPECIALIST 1 & 2**  **SEMESTER 1 2019**  **TEST 2**  **Calculator Assumed** |

Reading Time: 2 minutes

Time Allowed: 26 minutes Total Marks: 25

Question 6 (9 marks)

Points O, P, Q and R have position vectors , ,  and .

(a) Determine the value of y if . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes magnitude equation  ✓ states both +ve and -ve solutions |

(b) Determine the value of x if is parallel to . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines  ✓ solves for k  ✓ solves for x |

(c) Determine the values of x and y if R lies on the line between P and Q such that . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes vector equation  ✓ substitutes position vectors  ✓ solves for x using i-coefficients  ✓ solves for y using j-coefficients |

**Question 7 (10 marks)**

Two tug boats are acting on a oil rig in a harbour where there is a strong tidal flow. The tug boat force vectors **F1**= 4000N and **F2** =5000Nare shown in the diagram.

(a) given the tidal flow exerts a force **F3**= 3000N on a bearing 170o T:(1 mark)

draw the vector for ***F3***on the diagram

|  |  |
| --- | --- |
| Solution | |
|  | |
| Marking key/mathematical behaviours | Marks |
| * draws  in approximately the right direction (on a bearing of ~170 degrees) | 1 |

**Question 7 (b)** (5 marks)

|  |  |  |
| --- | --- | --- |
| Solution | | |
|  |  | |
| Marking key/mathematical behaviours | | Marks |
| * defines each vector correctly * gives the correct components of the total | | 3  2 |

**Question 7 (c)** (4 marks)

|  |  |  |
| --- | --- | --- |
| Solution | | |
|  |  | |
| Marking key/mathematical behaviours | | Marks |
| * calculates the magnitude correctly * calculates the angle the vector makes with the X-axis * gives the correct bearing * gives the results to an appropriate level of accuracy | | 1  1  1  1 |

Question 8 (6 marks)

An aircraft is to be flown directly from A to B, where  km. A steady wind with velocity km/h is blowing. Determine the velocity the aircraft should steer in the form , given that the aircraft has a cruising speed of 420 km/h.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes vector equation  ✓ writes  using i-coefficients  ✓ writes  using j-coefficients  ✓ writes speed equation  ✓ solves for x and y  ✓ states velocity vector |